ISSN 2229-5518

g-CLOSED TYPE SETS AND g*-CLOSED TYPE SETS IN TOPOLOGICAL ORDERED SPACES

G.SRINIVASARAO[1], N.SRINIVASARAO[2], CH. RAMPRASAD[3], M.V.SUBBARAO[4]

[1]*Tirumala Engineering College, Jonnalagadda(Vill.),Narasaraopet*
mohanadurga2000@gmail.com)
[2]*Vignan's University, Vadlamudi(Vill.), Guntur*
srinivasunimmaa@yahoo.co.in)
[3]*Vingnan's University, Vadlamudi(Vill.), Guntur*
rm.prasad3@gmail.com
[4]*Vignan's Universisty, Vadlamudi(Vill.), Guntur.*
subbaraovu@gmail.com

Abstract: - In this paper we introduce ig-closed sets, dg-closed sets , bg-closed sets, and studied relationship between them. Also we introduce ig*-closed sets, dg*-closed sets , bg*-closed sets and we discuss the possible relations between newly introducing sets.

Keywords: *- bg-closed, bg*-closed, dg-closed, dg*-closed, ig-closed and ig*-closed sets.*

I. INTRODUCTION

Leopoldo Nachbin [1] initiated the study of topological ordered spaces. Levine [4] introduced the class of g-closed sets, a super class of sets in 1970. M.K.R.S.Veera Kumar [2]introduced a new class of sets, called g*-closed sets in 2000, which is properly placed in between the class of closed sets and the class of g-closed sets. M.K.R.S.Veera Kumar [3] introduced the study of i-closed, d-closed and b-closed sets in 2001.

A topological ordered space is a triple $(X, \tau, \leq)$, where τ is a topology on X , Where X is a non-empty set and $\leq$ is a partial order on X.

DEFINITION 1.1 [3] For any $x \in X$, $\{y \in X / x \leq y\}$ will be denoted by $[x, \rightarrow]$ $\{y \in X / y \leq x\}$ will be denoted by $[\leftarrow, x]$. A subset A of a topological ordered space $(X, \tau, \leq)$ is said to be increasing if $A = i(A)$ where $i(A) = \bigcup_{a \in A} [a, \rightarrow]$.

DEFINITION 1.2 [3] For any $x \in X$, $\{y \in X / y \leq x\}$ will be denoted by $[\leftarrow, x]$. A subset A of a topological ordered space $(X, \tau, \leq)$ is said to be a decreasing if $A = d(A)$, where $d(A) = \bigcup_{a \in A} [a, \leftarrow]$

The complement of a decreasing (resp.an increasing) set is an increasing (resp. a decreasing) set. C(A) denotes the complement of A in X.

$icl(A) = \bigcap \{F/F$ is an increasing closed subset of X containing A$\}$

$dcl(A) = \bigcap \{F/F$ is a decreasing closed subset of X containing A$\}$

$bcl(A) = \bigcap \{F/F$ is a closed subset of X containing A with $F = i(F) = d(F)\}$

IO(X) (resp.**DO(X), BO(X))** denotes the collection of all increasing (resp.decreasing, both increasing and decreasing) open subsets of a topological ordered space $(X, \tau, \leq)$.

For a subset A of a space $(X, \tau, \leq)$, icl(A) (resp.dcl(A), bcl(A)) denote the increasing (resp.decreasing, both increasing and decreasing) closure of A.

ISSN 2229-5518

DEFINITION 2.1. A subset A of a topological space (X , τ) is called

1. a generalized closed set (briefly g-closed) [4] if cl(A)$\subseteq$ U whenever A$\subseteq$ U and U is open in (X, τ).
2. a g^*-closed set [1] if if cl(A)$\subseteq$ U whenever A$\subseteq$U and U is g-open in (X, τ).
3. an i-closed set [3] if A is an increasing set and closed set.
4. a d-closed set [3] if A is a decreasing set and closed set.
5. a b-closed set [3] if A is a both increasing and decreasing set and a closed set.

THEOREM 2.2. [2] Every closed set is a g-closed set.
The following example supports that a g-closed set need not be closed set in general.

EXAMPLE 2.3. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , $\tau_2,\leq_1$) is a topological ordered space.

closed sets are ϕ , X , {b , c}. g-closed sets are ϕ, X , {b}, {c}, {a,b}, {b , c}, {c,a}.

Let A={c}. Clearly A is a g-closed set but not a closed set.

THEOREM 2.4. [2] Every g^*-closed set is a g-closed set.

The following example supports that a g-closed set need not be a g^*-closed set in general.

EXAMPLE 2.5. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , $\tau_2,\leq_1$) is a topological ordered space.

g-closed sets are ϕ , X , {b}, {c}, {a,b}, {b , c}, {c,a}.

g^*-closed sets are ϕ, X , {b , c}.

Let A={c}. Then A is a g-closed set but not a g^*-closed set.

II. HEADINGS

§ 3. Results between ig, dg and bg-closed type sets

We introduce the following definitions.

DEFINITION 3.1. A subset 'A' of (X , τ, $\leq$) is called ig-closed set if icl(A)$\subseteq$U whenever A$\subseteq$ U and U is open in (X , τ).

The class of all ig-closed subsets of (X , τ, $\leq$) is denoted by IGC(X).

DEFINITION 3.2. A subset 'A' of (X , τ, $\leq$) is called a dg-closed set if dcl(A) $\subseteq$ U whenever A $\subseteq$U and U is an open in (X , τ).

The class of all dg-closed subsets of (X , τ) is denoted by DGC(X).

DEFINITION 3.3. A subset 'A' of (X , τ, $\leq$) is called a bg-closed set if bcl(A) $\subseteq$ U whenever A $\subseteq$U and U is an open in (X , τ).

The class of all bg-closed subsets of (X , τ) is denoted by BGC(X).

EXAMPLE 3.4. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , τ_1, $\leq_1$) is a topological ordered space.

Let A = {c}. A is an ig-closed set. Let B = {b}. B is not an ig-closed set.

THEOREM 3.5. Every i-closed set is an ig-closed set.

Proof. We know that every closed set is a g-closed set. Then every i-closed set is an ig-closed set.

ISSN 2229-5518

The following example supports that an ig-closed set need not be an i-closed set in general.

EXAMPLE 3.6. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_2, $\leq_2$) is a topological ordered space.

ig-closed sets are Φ , X , {b} , {a , b}. i-closed sets are ϕ , x. Let A = {b} or {a , b}.

Clearly A is an ig-closed set but not an i-closed set.

So the class of all ig-closed sets properly contains the class of all i-closed sets.

We introduce the following definition.

EXAMPLE 3.7. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_1, $\leq_3$) is a topological ordered space.

Let A = {a , c}. Clearly A is a dg-closed set. Let B = {a}. Clearly B is not a dg-closed set.

THEOREM 3.8. Every d-closed set is a dg-closed set.

Proof. We know that every closed set is a g-closed set. Then every d-closed set is a dg-closed set.

The following example supports that a dg-closed set need not be d-closed set in general.

EXAMPLE 3.9. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_2, $\leq_2$) is a topological ordered space.

dg-closed sets are ϕ , X , {c} , {b , c}. d-closed sets are ϕ, X , {b , c}.

Let A = {c}. Clearly A is a dg-closed set but not a d-closed set.So the class of all dg-closed sets properly contains the class of all d-closed sets.

EXAMPLE 3.10. Let X = {a , b , c} , τ_5 = { ϕ , X , {a} , {a , b} , {a , c}} and $\leq_5$ = {(a , a) , (b , b) , (c , c) , (a , c) , (b , c)}. Clearly (X , τ_5, $\leq_5$) is a topological ordered space.

Let A = {c}. Clearly A is a bg-closed set. Let B = {a , c}. Clearly B is not a bg-closed set.

THEOREM 3.11. Every b-closed set is a bg-closed set.

Proof. We know that every closed set is a g-closed set. Then every b-closed set is a bg-closed set.

The following example supports that a bg-closed set need not be a b-closed set in general.

EXAMPLE 3.12. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and

$\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_2, $\leq_3$) is a topological ordered space. bg-closed sets are ϕ , X , {c}. b-closed sets are ϕ , X.

Let A = {c}. Clearly A is a bg-closed set but not a b-closed set.

So the class of all bg-closed sets properly contains the class of all b-closed sets.

THEOREM 3.13. Every bg-closed set is an ig-closed set.

Proof. We know that every balanced set is an increasing set. Then every bg-closed set is an ig-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 3.14. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , τ_1, $\leq_1$) is a topological ordered space.

Let A = {c}. Clearly A is an ig-closed set but not a bg-closed set.

ISSN 2229-5518

THEOREM 3.15. Every bg-closed set is a dg-closed set.

Proof. We know that every balanced set is a decreasing set. Hence every bg-closed set is

a dg-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 3.16. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , $\tau_1, \leq_3$) is a topological ordered space.

Let A = {a , c}. Clearly A is a dg-closed set but not a bg-closed set.

The class of all dg-closed sets properly contains the class of all bg-closed sets.

THEOREM 3.17. ig-closedness and dg-closedness are independent notions. This will be proved by in the following examples.

EXAMPLE 3.18. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , $\tau_1, \leq_1$) is a topological ordered space. Let A = {c}. Clearly A is an ig-closed set but not a dg-closed set.

EXAMPLE 3.19. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , $\tau_1, \leq_3$) is a topological ordered space.

Let A = {a , c}. Clearly A is a dg-closed set but not an ig-closed set.

THEOREM 3.20. Every b-closed set set is an i-closed set.

Proof. We know that every balanced set is an increasing set. Then every b-closed set is an i-closed set.

. The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 3.21. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , $\tau_1, \leq_1$) is a topological ordered space.

i-closed sets are ϕ , X , {c} , {b , c}. b-closed sets are ϕ , X.

Let A = {c} or {b , c} . Clearly A is an i-closed set but not a b-closed set.

The class of all i-closed sets properly contains the class of all b-closed sets.

THEOREM 3.22. Every b-closed set is a d-closed set.

PROOF. We know that every balanced set is a decreasing set. Then every b-closed set is a d-closed set.

The converse of above theorem need not be true. This will be justify from the following example..

EXAMPLE 3.23. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , $\tau_1, \leq_2$) is a topological ordered space.

d-closed sets are ϕ , X , {c} , {b , c}. b-closed sets are ϕ , X.

Let A = {c} or {b , c}. Clearly A is a d-closed set but not a b-closed set.

The class of all d-closed sets properly contains the class of all b-closed sets.

THEOREM 3.24. i-closedness and d-closedness are independent notions. This will be proved by the following examples.

ISSN 2229-5518

EXAMPLE 3.25. Example 3.21 shows that A = {c} or {b , c} is an i-closed set but not a d-closed set.

EXAMPLE 3.26. Example 3.23 shows that A = {c} or {b , c} is a d-closed set but not a i-closed set.

§4. Results between ig^*, dg^* and bg^*-closed type sets

We introduce the following definition.

DEFINITION 4.1. A subset 'A' of (X , τ, $\leq$) is called a ig^*-closed set if icl(A) $\subseteq$ U whenever A $\subseteq$U and U is a g-open in (X , τ).

The class of all ig^*-closed subsets of (X , τ) is denoted by $IG^*C(X)$.

DEFINITION 4.2. A subset 'A' of (X , τ, $\leq$) is called a dg^*-closed set if dcl(A) $\subseteq$ U whenever A $\subseteq$U and U is a g-open in (X , τ).

The class of all dg^*-closed subsets of (X , τ) is denoted by $DG^*C(X)$.

DEFINITION 4.3. A subset 'A' of (X , τ,$\leq$) is called a bg^*-closed set if bcl(A) $\subseteq$ U whenever A $\subseteq$U and U is a g-open in (X , τ).

The class of all dg^*-closed subsets of (X , τ) is denoted by $BG^*C(X)$.

THEOREM 4.4. Every ig^*-closed set is an ig-closed set.

Proof. We know that every g^*-closed set is a g-closed set. Then every ig^*-closed set is an ig-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.5. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , τ_2, $\leq_1$) is a topological ordered space.

ig-closed sets are ϕ , X , {c} , {b , c}. ig^*-closed sets are ϕ, X , {b , c}.

Let A = {c}. Clerly A is an ig-closed set but not a ig^*-closed set.

So the class of all ig-closed sets properly contains the class of all ig^*-closed sets.

THEOREM 4.6. Every dg^*-closed set is an dg-closed set.

Proof. We know that every g^*-closed set is a g-closed set. Then every dg^*-closed set is an dg-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.7. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_2, $\leq_2$) is a topological ordered space.

dg-closed sets are ϕ , X , {c} , {b , c}. dg^*-closed sets are ϕ , X , {b , c}.

Let A = {c}. Clerly A is an dg-closed set but not a dg^*-closed set.

So the class of dg-closed sets properly contains the class of all dg^*-closed sets.

THEOREM 4.8. Every bg^*-closed set is a bg-closed set.

Proof. We know that every g^*-closed set is a g-closed set. Then every bg^*-closed set is a bg-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.9. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) ,

ISSN 2229-5518

(a , c)}.Clearly (X , $\tau_2, \leq_3$) is a topological ordered space.

bg*-closed sets are ϕ , X. bg-closed sets are ϕ , X , {c}.

Let A = {c}. Clearly A is bg-closed set but not a bg*-closed set.

So the class of bg-closed sets properly contains the class of all bg*-closed sets.

THEOREM 4.10. Every bg*-closed set is an ig*-closed set.

Proof. We know that every balanced set is an increasing set. Then every bg*-closed set is an ig*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.11. Let X = {a , b , c} , τ_3 = { ϕ , X , {a} , {b , c}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , $\tau_3, \leq_3$) is a topological ordered space.

Let A = {b}. Clearly A is an ig*-closed set but not a bg*-closed set.

THEOREM 4.12. Every bg*-closed set is an dg*-closed set.

Proof. We know that every balanced set is an dereasing set. Then every bg*-closed set is an dg*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.13. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , $\tau_1, \leq_3$) is a topological ordered space.

Let A = {a , c}. Clearly A is a dg*-closed set but not a ig*-closed set.

The class of all dg*-closed sets properly contains the class of all bg*-closed sets.

THEOREM 4.14. ig*-closedness and dg*-closedness are independent notions. This will be proved by in the following examples.

EXAMPLE 4.15. Let X = {a , b , c} , τ_3 = { ϕ , X , {a} , {b , c}} and $\leq_3$ = {(a , a) , (b , b), (c , c) , (a , b) , (a , c)}. Clearly (X , $\tau_3, \leq_3$) is a topological ordered space.Let A = {b}. Clearly A is an ig*-closed set but not a dg*-closed set.

EXAMPLE 4.16. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , $\tau_1, \leq_3$) is a topological ordered space.

Let A = {a , c}. Clearly A is a dg*-closed set but not a ig*-closed set.

THEOREM 4.17. Every i-closed set is an ig*-closed set.

Proof. We know that every closed set is a g*-closed set. Then every i-closed set is an ig*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.18. Let X = {a , b , c} , τ_3 = { ϕ , X , {a} , {b , c}} and $\leq_4$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , $\tau_3, \leq_4$) is a topological ordered space.

ig*-closed sets are ϕ , X , {b , c}. i-closed sets are ϕ , X.

Let A = {b , c}. Clearly A is a ig*-closed set but not an i-closed set.

The class of all ig*-closed sets properly contains the class of all i-closed sets.

THEOREM 4.19. Every d-closed set is a dg*-closed set.

Proof. We know that every closed set is a g*-closed set. Then every d-closed set is a dg*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

ISSN 2229-5518

EXAMPLE 4.20. Let X = {a , b , c} , τ_4 = { ϕ , X , {a} , {b , c}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_4, $\leq_2$) is a topological ordered space.

dg*-closed sets are ϕ , X , {b , c}. d-closed sets are ϕ , X.

Let A = {b , c}. Then A is dg*-closed set but not a d-closed set.

The class of all dg*-closed sets properly contains the class of all d-closed sets.

THEOREM 4.21. Every b-closed set is a bg*-closed set.

PROOF. We know every closed set is a g*-closed set. Then every b-closed set is a bg*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.22. Let X = {a , b , c} , τ_6 = { ϕ , X , {a} , {b} , {a , b}{a , c}} and $\leq_7$ = {(a , a), (b , b), (c , c), (b , c), (c , a), (b , a)}. Clearly (X , τ_6, $\leq_7$) is a topological ordered space.bg*-closed sets are ϕ , X , {b}. b-closed sets are ϕ , X.

Let A = {b}. Then A is bg*-closed set but not a b-closed set.

The class of all bg*-closed sets properly contains the class of all b-closed sets.

THEOREM 4.23. Every bg*-closed set is an ig-closed set.

Proof. We know that every balanced set is an increasing set and every g*-closed set is a g-closed set . Then every bg*-closed set is an ig-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.24. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_1, $\leq_3$) is a topological ordered space.

bg*-closed sets are ϕ , X. ig-closed sets are ϕ , X , {c} , {b , c}.

Let A = {c} or {b , c}. Clearly A is an ig-closed set but not a bg*-closed set.

The class of all ig-closed sets properly contains the class of all bg*-closed sets.

THEOREM 4.25. Every bg*-closed set is a dg-closed set.

Proof. We know that every balanced set is a decreasing set and every g*-closed set is a g-closed set . Then every bg*-closed set is a dg-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.26. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}.Clearly (X , τ_1, $\leq_2$) is a topological ordered space.

bg*-closed sets are ϕ , X. dg-closed sets are ϕ , X , {c} , {b , c}.

Let A = {c} or {b , c}. Clearly A is a dg-closed set but not a bg*-closed set.

The class of all dg-closed sets properly contains the class of all bg*-closed sets.

THEOREM 4.27. bg-closedness and ig*-closedness are independent notions. This will be seen in the following examples.

EXAMPLE 4.28. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , τ_1, $\leq_1$) is a topological ordered space.

ig*-closed sets are ϕ , X , {c} , {b , c}. bg-closed sets are ϕ , X

Let A = {c} or {b , c}. Clearly A is an ig*-closed but not a bg-closed set.

ISSN 2229-5518

EXAMPLE 4.29. Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_5$ = {(a , a) , (b , b) , (c , c) , (a , c) , (b , c)}. Clearly (X , τ_2, $\leq_5$) is a topological ordered space.ig*-closed sets are ϕ , X , {b , c}. bg-closed sets are ϕ , X , {c} , {b , c} , {c , a}.

Let A = {c} or {c , a}. Clearly A is an bg-closed set but not a ig*-closed set.

THEOREM 4.30. bg-closedness ad dg*-closedness are independent notions. This will be seen in the following examples.

EXAMPLE 4.31. . Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_1, $\leq_3$) is a topological ordered space.

dg*-closed sets are ϕ , X , {c} , {b , c}. bg-closed sets are ϕ , X , {c}.

Let A = {b , c}. Clearly A is a dg*-closed set but not a bg-closed set.

EXAMPLE 4.32. . Let X = {a , b , c} , τ_2 = { ϕ , X , {a}} and $\leq_3$ = {(a , a) , (b , b) , (c , c) , (a , b) , (a , c)}. Clearly (X , τ_2, $\leq_3$) is a topological ordered space.

dg*-closed sets are ϕ , X. bg-closed sets are ϕ , X , {c}.

Let A = {c}. Clearly A is a bg-closed set but not a dg*-closed set.

THEOREM 4.33. Every i-closedness and bg*-closedness are independent notions. This will be seen in the following examples.

EXAMPLE 4.34. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , τ_1, $\leq_1$) is a topological ordered space.

bg*-closed sets are ϕ , X. i-closed sets are ϕ , X , {c} , {b , c}.

Let A = {c} or {b , c}. Clearly A is an i-closed set but not a bg*-closed set.

The class of all i-closed sets properly contains the class of all bg*-closed sets.

EXAMPLE 4.35. Let X = {a , b , c} , τ_{10} = { ϕ , X , {c} , {b , c}} and $\leq_5$ = {(a , a) , (b , b) , (c , c) , (b , c) , (a , c)}. Clearly (X , τ_{10}, $\leq_5$) is a topological ordered space. bg*-closed sets are ϕ , X , {c , a}. i-closed sets are ϕ , X.

Let A = {c , a}. Clearly A is a bg*-closed set but not an i-closed set.

The class of all bg*-closed sets properly contains the class of all i-closed sets.

THEOREM 4.36. d-closedness and bg*-closedness are independent notions. This will be seen in the following examples.

EXAMPLE 4.37. Let X = {a , b , c} , τ_1 = { ϕ , X , {a} , {b} , {a , b}} and $\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b))}. Clearly (X , τ_1, $\leq_2$) is a topological ordered space. bg*-closed sets are ϕ , X. d-closed sets are ϕ , X , {c} , {b , c}.

Let A = {c} or {b , c}. Clearly A is a d-closed set but not a bg*-closed set.

The class of all bg*-closed sets properly contains the class of all d-closed sets.

EXAMPLE 4.38. Let X = {a , b , c} , τ_8 = { ϕ , X , {a , b}} and $\leq_5$ = {(a , a) , (b , b) , (c , c) , (a , c) , (b , c))}. Clearly (X , τ_8, $\leq_5$) is a topological ordered space.

bg*-closed sets are ϕ , X , {c} , {b , c} , {c , a}. d-closed sets are ϕ , X , {c}.

Let A = {b , c} or {c , a}. Clearly A is bg*-closed set but not a d-closed set.

The class of all bg*-closed sets properly contains the class of all d-closed sets.

ISSN 2229-5518

THEOREM 4.39. Every b-closed set is an ig*-closed set.Proof. We know that every closed set is a g*-closed set and every balanced set is an increasing set. Then every b-closed set is an ig*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.40. Let X = {a , b , c} , τ_1 = {ϕ , X , {a} , {b} , {a , b}} and $\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}. Clearly (X , τ_1, $\leq_1$) is a topological ordered space.

ig*-closed sets are ϕ , X , {c} , {b , c}. b-closed sets are ϕ , X.

Let A = {c} or {b , c}. Clearly A is an ig*-closed set but not a b-closed set.

The class of ig*-closed sets properly contains the class of all b-closed sets.

THEOREM 4.41. Every b-closed set is a dg*-closed set.

Proof. We know that every balanced set is a decreasing set and every closed set is a g*-closed set. Then every b-closed set is a dg*-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.42. Let X = {a , b , c} , τ_1 = {ϕ , X , {a} , {b} , {a , b}} and

$\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}.Clearly (X , τ_1, $\leq_2$) is a topological ordered space. dg*-closed sets are ϕ, X , {c} , {b , c}. b-closed sets are ϕ , X.

Let A = {c} or {b , c}. Clearly A is a dg*-closed set but not a b-closed set.

The class of dg*-closed sets properly contains the class of all b-closed sets.

THEOREM 4.43. Every b-closed set is an ig-closed set.

Proof. We know that every closed set is a g-closed set and every balanced set is an increasing set. Then every b-closed set is an ig-closed set.

The converse of the above theorem need not be true as we see the following example.

EXAMPLE 4.44. Let X = {a , b , c} , τ_1 = {ϕ , X , {a} , {b} , {a , b}} and

$\leq_1$ = {(a , a) , (b , b) , (c , c) , (a , b) , (b , c) , (a , c)}.Clearly (X , τ_1, $\leq_1$) is a topological ordered space. ig-closed sets are ϕ , X , {c} , {b , c}. b-closed sets are ϕ , X.

Let A = {c} or {b , c}. Clearly A is an ig-closed set but not a b-closed set.

The class of all ig-closed sets properly contains the class of all b-closed sets.

THEOREM 4.43. Every b-closed set is a dg-closed set.

Proof. We know that every balanced set is a decreasing set and every closed set is a g*-closed set. Then every b-closed set is a dg-closed set.

The converse of above theorem need not be true. This will be justify from the following example.

EXAMPLE 4.44. Let X = {a , b , c} , τ_1 = {ϕ , X , {a} , {b} , {a , b}} and

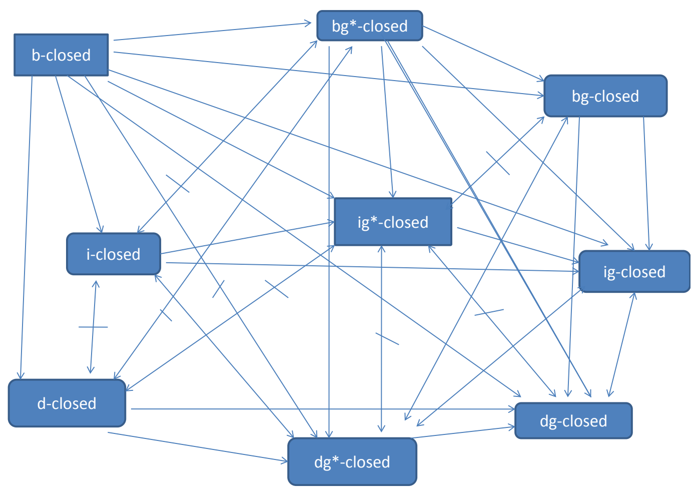
$\leq_2$ = {(a , a) , (b , b) , (c , c) , (a , b) , (c , b)}. Clearly (X , τ_1, $\leq_2$) is a topological ordered space. dg-closed sets are ϕ , X , {c} , {b , c}. b-closed sets are ϕ , X.

Let A = {c} or {b , c}. Clearly A is a dg-closed set but not a b-closed set.

III.

IV.

V. FIGURES AND TABLES



VI. CONCLUSION

In this paper, we introduced , new class of sets, studied various relationship between them.

ACKNOWLEDGEMENTS

An acknowledgement section may be presented after the conclusion, if desired.

REFERENCES

1. **Leopoldo Nachbin, Topology and order, D.Van Nostrand Inc., Princeton, New Jersey [1965].**

2. **M.K.R.S. Veera Kumar, Between closed sets and g-closed sets, Mem.Fcc.Sci.Sec A , Math.Kochi University, 21(2000), 1-19.**

3. **M.K.R.S. Veera Kumar, Homeomorphisms in topological ordered spaces, Acta Ciencia Indica, XXVIII(M), No.1.(2002), 67-76.**

4. **N.Levine, Generalized closed sets in topology, Rend. Circ.Math.Palermo, 19(2) (1970), 89-96.**